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Article

Gravitational waves in f(Q) gravity

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ABSTRACT

The evolution of gravitational waves (GWs) in *f(Q)* gravity, whose foundation is the non-metricity (NM) scalar *Q*, has been examined in this study. We discuss and study the dynamics of the background evolution of GWs in a flat FRW universe. Two distinct *f(Q)* toy models are considered for the work. The perturbed equations controlling the evolution of GWs in terms of redshift *z* have been derived in the FRW Universe backdrop using the field equations. We then explored the properties of GWs within the paradigm of *f(Q)* gravity and obtained some intriguing conclusions.

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I INTRODUCTION

For the past twenty years, the late cosmic acceleration has been a contentious topic of discussion. Recent advances in observational cosmology, including large-scale structure observations (Koivisto and Mota 2006, Daniel 2008, Nadathur et al 2020), Cosmic Microwave Background Radiation (CMBR) (Spergel et al 2003), and type Ia supernovae (Perlmutter et al 1999, Riess et al 1998, Riess et al 2004), confirm this feature of the universe. However, the real issue lies in the theory being worked out to account for this universe-processing machinery. For more than a century, general relativity (GR) has been widely acknowledged as the most successful fundamental gravitational theory for explaining the large-scale structure of the universe. The matter source and the FRW spacetime in cosmology provide exact solutions for the scale factor *a(t)*, which helps us comprehend the universe's expansion. The universe needs to go through two different kinds of rapid expansion in order to recover FRW spacetime. The first one is the early inflationary universe, which can be studied by supplementing the Einstein-Hilbert action with a scalar field. Furthermore, the inclusion of a cosmological constant into the Einstein field equation (EFE) allows for the explanation of the present rapid expansion. The universe's late-time cosmic acceleration can be explained by this very simple model, called the ΛCDM model, which fits the observational data. Unfortunately, this beautiful model is restricted by the problems with vacuum energy (Copeland et al 2006, Nojiri and Odintsov 2007) fine-tuning. As such, it is imperative to consider alternative approaches or generalizations of fundamental theories of gravity. The simplest generalization of general relativity is known as f(R) gravity (Buchdahl 1970, Barrow and Ottewill 1983), which expands the Einstein-Hilbert (EH) action to a general function of the Ricci scalar R. This modified theory of gravity (MTG) is widely known for its successful description of cosmic acceleration, for reproducing the complete history of the universe and the behavior of the cosmological constant Λ (Nojiri and Odintsov 2006, Elizalde and Gomez 2009, la Cruz and Dobado 2006). Various methods have been employed in the literature to employ MTG to retrieve the attributes of ΛCDM. *f(R, T)* (Harko et al 2011) and f(R, T2) (Katirci and Kavuk 2014, Roshan and Shojai 2016) theories, where T is the trace of the stress energy tensor or the energy-momentum tensor Tμν and T 2 = $T\mu\nu T\ \mu\nu$, were developed as a result of additional changes to f(R) theories. These concepts consider a non-zero interplay between the geometry and matter components, leading to intriguing phenomenologies. All the previously suggested extensions of GR share a common trait, which is the underlying Riemannian geometry (Riemann 1854) (formulated in the Riemann metrical space) at the heart of all these classical theories, including GR. Considering the incompatibility of these theories at certain scales, it is reasonable to ask whether any of the long-standing conflicts in these classical theories may be resolved if a much more universal geometric framework were to replace the underlying geometry. Such a daring endeavor was made by Weyl (Weyl 1918), whose main objective was the mathematical unification of gravity and electromagnetism. We know that the basic instrument for contrasting vector lengths in the geometry of curved space is the Levi-Civita connection and it is compatible with the metric. The theory proposed by Weyl makes use of a brand-new technique that involves two connections: the first one controls the direction of a vector during parallel transit and the other one conveys its length data. The most notable feature of this is the non-vanishing covariant derivative of the fundamental tensor, which results in non-metricity (NM) Q, a new geometric quantity. Weyl's geometry is a mathematical masterpiece with a correspondingly a complex physical framework. Two fundamental formulations of gravity are found in the literature: the teleparallel $(R = 0, T \neq 0)$ conceptualization, where T is the scalar torsion, and the curvature $(R \neq 0, T = 0)$ conceptualization. But in these two conceptualizations, the NM scalar Q becomes zero. In case of geometry, Q represents the variance in a vector's length while it is transported parallelly. Currently, under a third comparable GR formalism, a non-zero non-metricity Q was believed to represent the elementary geometric parameter necessary for all sorts of gravitational interconnections. This theory is known as symmetric teleparallel gravity (STG) (Nester and Yo 1999). Here the energy-momentum density is denoted by the Einstein pseudotensor. In the geometric representation, this pseudotensor ultimately becomes a real tensor. Other names for f(Q) gravity (Jimenez et al 2018) are coincident GR and non-metric gravity. Cosmology of f(Q) gravity and its empirical constraints were studied in (Lu et al 2019, Lazkoz et al 2019). In recent decades, there have been multiple advancements made to the STG framework (Adak et al 2006, Adak 2006, Adak et al 2013, Mol 2017, Harko et al 2018). According to Harko et al 2018, the non-minimal coupling connecting the NM scalar Q and the Lagrangian of matter Lm should be scrutinized when expanding the f(Q) gravity. The non-minimal connection bridging the geometry and matter parts leads to relatively expected consequences, including the violation of conservation of the EMT and the inclusion of an extra force in the motion equation of geodesic. Xu et al. suggested the *f(Q,* T) gravity as an additional generalized form of the theory in (Xu et al 2019). In this case, the trace of the EMT T and Q are essentially arbitrary functions that define the gravity Lagrangian. The cosmic evolution of the model was analyzed and the field equations were found. Both physicists and astronomers have been fascinated with gravitational waves (GWs), a remarkable ramification of Einstein's GR. GWs are critical aspects of cosmology and astrophysics. Measurements of polarization and cosmic microwave temperature anisotropy can be utilized to obtain important data regarding the amplitude of these spacetime ripples (Crittenden et al 1993a, Crittenden et al 1993b). It seems reasonable that perturbations in the gravitational field could spread like waves since this is how other kinds of waves that we see in nature behave. The relationship between gravitational waves and the core ideas of general relativity was further cemented when Einstein proved that gravitational radiation, or GWs, is a natural consequence of his theory. Einstein's field equations provide a linear wave equation with plane wave solutions in the limit of tiny deviations from Euclidean space-time (or Minkowski space). These solutions characterize transverse metric perturbations of Minkowski space that have properties similar to EM waves, and that propagate at the speed of light. Although there are many parallels between EM waves and GWs, it is important to remember that there are also differences, therefore this comparison should be used with caution. GWs separated from matter in the early Universe as a result of gravitons decoupling. As such, these GWs are essential for constraining and differentiating cosmological parameters in different cosmological theories. Interestingly, vacuum fluctuations gave rise to primordial GWs. A cosmological model that seeks to unify dark energy (DE) and dark matter (DM), the two dark sectors of the Universe, can describe the matter content of a hypothetical early Universe. This is where a smooth changeover takes place between a radiation-dominated epoch and an early de Sitter-like phase. Given all the possible DE models in cosmology, GWs can shed light on the periods when the underlying cosmic dynamics fluctuated. As a result, *GW*s have been the subject of numerous studies of late (Buonanno et al 1997, Gasperini 1997, Fabris et al 1998, Infante and Sanchez 2000, Riazuelo and Uzan 2000, Fabris et al 2004, Santos et al 2005, Lopez et al 2010, Zhang and Cheung 2020, Debnath 2020, Debnath 2021). Herein, our primary objective is to explore the characteristics of GWs within the realm of *f(Q)* gravity in a Friedmann Robertson-Walker (FRW) spacetime. In order to achieve this, we study how the dynamics of GWs are affected by *f(Q)* gravity. We investigate the *GW* evolution in a flat FRW universe taking the intricacies of *f(Q)* gravity into consideration. The work is structured as follows: Background equations of *f(Q)* gravity are studied in section II. The dynamics of the GWs in the context of *f(Q)* gravity are explored in section III. In section IV, the study concludes with a brief statement.

II. BASIC EQUATIONS OF f(Q) GRAVITY

We will go over the fundamental ideas and associated equations of $f(Q)$ gravity in this part. We start with the following action,

$$S=\int\left[-\frac{1}{2\kappa^2}f(Q)+\mathcal{L}_m\right]\sqrt{-g}d^4x \tag{1}$$

where $\mathcal{L}_m$ is the Lagrangian of matter component, $f(Q)$ is any function of the scalar Q and g is the determinant of the fundamental tensor $g_{\mu\nu}$. The NM scalar is defined as

$$Q=-\frac{1}{4}Q_{\alpha\beta\gamma}Q^{\alpha\beta\gamma}+\frac{1}{2}Q_{\alpha\beta\gamma}Q^{\gamma\beta\alpha}+\frac{1}{4}Q_\alpha Q^\alpha-\frac{1}{2}Q_\alpha\tilde{Q}^\alpha, \tag{2}$$

where

$$Q_\alpha\equiv Q^\mu_{\alpha\mu}, \tag{3}$$

$$\tilde{Q}^\alpha\equiv Q^{\mu\alpha}_\mu \tag{4}$$

and the NM tensor as,

$$Q_{\alpha\mu\nu}\equiv\nabla_\alpha g_{\mu\nu}. \tag{5}$$

When we take $f(Q)=Q$ then we derive the STG equivalent of GR. Now from the equations (1),(2), (3), (4), (5), the field equations are found as

$$\frac{2}{\sqrt{-g}}\nabla_\alpha\Big\{\sqrt{-g}g_{\beta\nu}f_Q\Big[-\frac{1}{2}L^{\alpha\mu\beta}+\frac{1}{4}g^{\mu\beta}(Q^\alpha-\tilde{Q}^\alpha)-\frac{1}{8}(g^{\alpha\mu}Q^\beta+g^{\alpha\beta}Q^\mu)\Big]\Big\}$$

$$+f_Q\Big[-\frac{1}{2}L^{\mu\alpha\beta}-\frac{1}{8}(g^{\mu\alpha}Q^\beta+g^{\mu\beta}Q^\alpha)+\frac{1}{4}g^{\alpha\beta}(Q^\mu-\tilde{Q}^\mu)\Big]Q_{\nu\alpha\beta}+\frac{1}{2}\delta^\mu_\nu f=\kappa^2T^\mu_\nu, \tag{6}$$

where $f_Q\equiv\frac{\partial f}{\partial Q}$. Here the deformation tensor is defined as

$$L^\alpha_{\mu\nu}=\frac{1}{2}Q^\alpha_{\mu\nu}-Q^\alpha_{\mu\nu} \tag{7}$$

and the matter EMT is

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m}{\delta g^{\mu\nu}} \tag{8}$$

The homogeneous and isotropic Friedmann-Lemaitre-Robertson-Walker (FLRW) metric is the subject of our next discussion. It may be given as

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{9}$$

where $a(t)$ is the cosmological scale factor with t denoting the cosmic time. Now, the field equations (6) of $f(Q)$ gravity (where $Q = 6H^2$) in the background of FRW spacetime is derived as

$$3H^2 = \kappa^2(\rho_m + \rho_Q), \tag{10}$$

$$3H^2 + \dot{H} = -\kappa^2(p_m + p_Q) \tag{11}$$

where H denotes the Hubble parameter which can be given as $H = \frac{\dot{a}}{a}$. The contributions of energy density and pressure from the modified gravity appearing in the FLRW equations are

$$\rho_Q = \frac{3}{\kappa^2}\Big[H^2(1 - 2f_Q) + \frac{f}{6}\Big], \tag{12}$$

$$p_Q = -\frac{1}{\kappa^2}\Big[2\dot{H}(1 - f_Q) + \frac{f}{2} + 3H^2(1 - 8f_{QQ}\dot{H} - 2f_Q\Big]. \tag{13}$$

A derivative with regard to the cosmic time t is represented by a dot in this instance. Moreover p_m is the matter fluid's pressure and ρ_m is the energy density of matter. The conservation equation of matter may be stated as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \tag{14}$$

III. GRAVITATIONAL WAVES

Here we will investigate the evolution of GWs in the f(Q) gravity framework. First, we discuss the basic equations that we need to develop the gravitational wave framework. Then we go on to study the characteristics of the gravitational waves generated in this set-up.

A. **Basic Equation**

For standard and flat FRW metric the field equations of $f(Q)$ gravity are given as (for $\kappa = 1$) (Debnath 2020)

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3}\left(\rho_m + \rho_Q\right), \tag{15}$$

$$\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} = -\left(p_m + p_Q\right), \tag{16}$$

where $a(t)$ is the cosmological scale factor. The energy density ρ_Q and pressure p_Q are given by the equations (12) and (13) respectively.

Given the assumption of separate conservation for DM and DE, the continuity equations for these components are given by

$$\dot{\rho}_m + \frac{3\dot{a}}{a}\left(\rho_m + p_m\right) = 0, \tag{17}$$

and

$$\dot{\rho}_Q + \frac{3\dot{a}}{a}\left(\rho_Q + p_Q\right) = 0. \tag{18}$$

Considering DM satisfies the equation of state (EoS) $p_m = \omega_m \rho_m$, where ω_m is the constant EoS parameter, we obtain

$$\rho_m = \rho_{m_0} a^{-3(1+\omega_m)} = \rho_{m_0} (1+z)^{3(1+\omega_m)}, \tag{19}$$

where ρ_{m_0} is the current value of the DM density and $z = \frac{a_0}{a} - 1$ is the cosmological redshift.
The governing equations of the evolution of gravitational waves for flat universe are as follows:

$$\ddot{\eta}(t) - \frac{\dot{a}}{a}\dot{\eta}(t) + \left(\frac{\xi^2}{a^2} - 2\frac{\ddot{a}}{a}\right)\eta(t) = 0, \tag{20}$$

which can be considered as,

$$\eta^{''}(z) - \left(\frac{\ddot{a}}{\dot{a}^2} - \frac{2}{a}\right)a^2\eta^{'}(z) + \frac{a^4}{\ddot{\eta}^2}\left(\frac{\xi^2}{a^2} - 2\frac{\ddot{a}}{a}\right)\eta(z) = 0 \tag{21}$$

where $'$ is the derivative with respect to redshift z.
Next we express the equations (15) and (16) as follows:

$$\frac{\dot{a}^2}{a^2} = H_0^2 X(z) \quad \text{and} \quad \frac{2\ddot{a}}{a} = -H_0^2 Y(z) \tag{22}$$

where

$$X(z) = \frac{1}{3H_0^2}(\rho_m + \rho_Q) \tag{23}$$

and

$$Y(z) = \frac{1}{3H_0^2}\left[\rho_m(1+3\omega_m) + \rho_Q(1+3\omega_Q)\right] \tag{24}$$

where $\omega_m = \frac{p_m}{\rho_m}$, $\omega_Q = \frac{p_Q}{\rho_Q}$ are the EoS parameter of DM and DE originating from the gravity modification respectively. H_0 is the current value of the Hubble parameter $H = \frac{\dot{a}}{a}$. Exploiting equations (22), (23) and (24), equation (21) reads

$$\eta^{''}(z) + \frac{1}{2(1+z)}\left(\frac{Y(z)}{X(z)} + 4\right)\eta^{'}(z) + \frac{4}{Y^2(z)H_0^4}\left(\xi^2 + \frac{H_0^2 Y(z)}{(1+z)^2}\right)\eta(z) = 0. \tag{25}$$

The characteristics of the gravitational waves for $f(Q)$ gravity in the background of a flat FLRW Universe are going to be investigated in the following sections.

B. Gravitational waves in *f (Q)* gravity

Here, the evolution of gravitational waves for various $f(Q)$ toy models is explored. The perturbation equation regulating the evolution of the gravitational waves will be framed and solutions will be sought. To do this we have to examine definite toy models of $f(Q)$ gravity which is done below.

1. Model 1

We examine the dynamic power-law $f(Q)$ function, which effectively depicts the universe's transition from a phase dominated by matter to the de-Sitter era given by (Shabani et al 2023),

$$f(Q) = \gamma\left(\frac{Q}{Q_0}\right)^n \tag{26}$$

where $Q_0 = 6H_0^2$, and γ, n are free parameters. Let us suppose the power-law form of the scale factor as:

$$a(t) = b_0 t^s \tag{27}$$

where b_0, s are constants. For these assumptions, the energy density and the pressure take the form

$$\rho_Q = 3s^2 b_0^{2/s}(z+1)^{2/s}\left(1 - \frac{6^{n-1}(1-2n)\gamma s^{2n-2} b_0^{\frac{2n-2}{s}}(z+1)^{\frac{2n-2}{s}}}{Q_0^n}\right) \tag{28}$$

$$p_Q = s b_0^{2/s}(z+1)^{2/s}\left(2 - 3s + 6^{n-1}\gamma\left(4(n-1)n + 9s^2\right)s^{2n-3} b_0^{\frac{2n-2}{s}}(z+1)^{\frac{2n-2}{s}} 3Q_0^n\right) \tag{29}$$

Using these in eqns.(23) and (24) and also using eqn.(19) we get the expressions for $X(z)$ and $Y(z)$ as,

$$X(z) = \frac{\left(s^2 b_0^{2/s}(z+1)^{2/s}\right)\left(1 - \frac{6^{n-1}(1-2n)\gamma s^{2n-2} b_0^{\frac{2n-2}{s}}(z+1)^{\frac{2n-2}{s}}}{Q_0^n}\right)^2}{H_0} + \Omega_{m0}(z+1)^{3(\omega_m+1)} \tag{30}$$

$$Y(z) = (3\omega_m + 1)\Omega_{m0}(z+1)^{3(\omega_m+1)} + \frac{1}{H_0^2}\left(s^2 b0^{2/s}(z+1)^{2/s}\right)\left(1 - \frac{6^{n-1}(1-2n)\gamma s^{2n-2} b0^{\frac{2n-2}{s}}(z+1)^{\frac{2n-2}{s}}}{Q_0^n}\right)$$

$$\left(\frac{-\frac{6^{n-1}\gamma\left(4(n-1)n+9s^2\right)s^{2n-3} b_0^{\frac{2n-2}{s}}(z+1)^{\frac{2n-2}{s}}}{3Q_0^n} - 3s + 2}{1 - \frac{6^{n-1}(1-2n)\gamma s^{2n-2} b_0^{\frac{2n-2}{s}}(z+1)^{\frac{2n-2}{s}}}{Q_0^n}} + 1\right) \tag{31}$$

Using the above two expressions in equation (25) we get a differential equation for $\eta(z)$. The trend of $\eta(z)$ is shown in Fig.(1). The figure shows the evolution of gravitational waves as z evolves. The plot is generated for two different values of ξ. The figure shows that gravitational waves of lower amplitude are obtained for a higher value of ξ. Moreover, with the evolution of time (as z becomes smaller) the waves proceed with decreasing amplitude.

2. *Model 2*

We now examine the recently presented $f(Q)$ model, which is similar to DGP formalism. It is given given by (Ayuso et al 2022),

$$f(Q) = \beta_1\sqrt{Q}\log Q + 2\beta_2 Q \tag{32}$$

where β_1 and β_2 are free parameters. This model is motivated by the need for an extra term that is equal to H in the modified Friedmann equation. Thus, taking into account $\sqrt{Q}$ in the model will result in an additional H component on

$$Y(z) = -\frac{1}{6H_0^2}\left(s b_0^{2/s}(z+1)^{2/s}\left(-2\sqrt{6}\beta_1\log\left(s b_0^{1/s}(z+1)^{1/s}\right) - \sqrt{6}\beta_1\log(6) + 6s(6\beta_2 + 9(4\beta_2+1)s - 7)\right) + \sqrt{6}\beta_1 s b_0^{1/s}(z+1)^{1/s}\right.$$

$$\left.\left(36s\log\left(s b_0^{1/s}(z+1)^{1/s}\right) + 14\log\left(\frac{b_0^{-1/s}(z+1)^{-1/s}}{s}\right) - 18s + 8 + \log(6)\right) - 6H_0^2(3\omega_m+1)\Omega_{m0}(z+1)^{3\omega_m+3}\right) \tag{36}$$

Using these two expressions in equation (25) we get a differential equation of $\eta(z)$ for model-II. The trajectory for $\eta(z)$ is given in Fig.(2). In the figure, the evolution of gravitational waves for this DGP-like $f(Q)$ model is obtained. The plot is generated for two different values of ξ. The figure shows that we get gravitational waves of lower amplitude for a higher value of ξ. Moreover, with the evolution of time (as z becomes smaller) the waves proceed with decreasing amplitude.

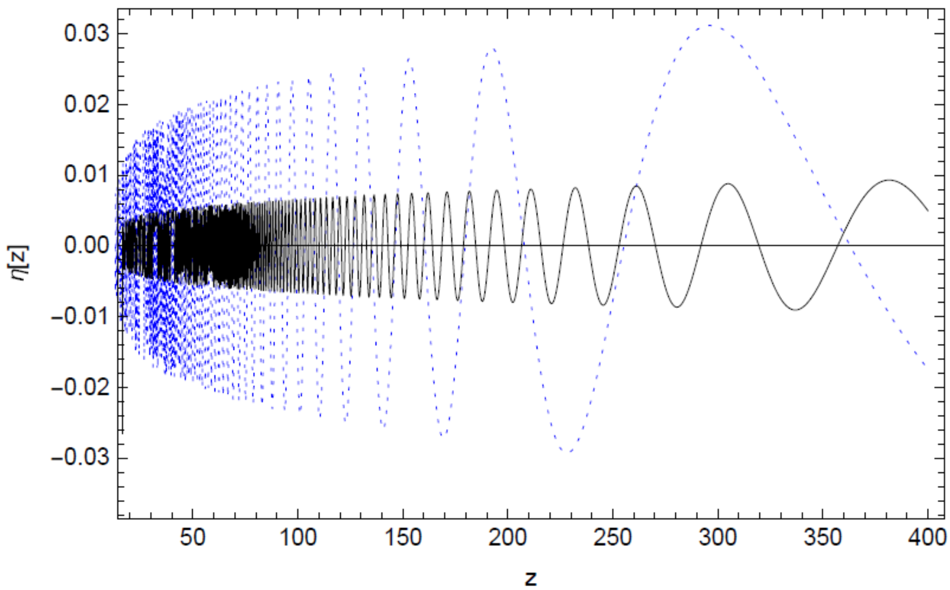

FIG. 1: The figure shows the variation of $\eta(z)$ against the redshift z, for model I. Here we have ξ = 5 × 1011 (black line) and ξ = 2 × 109 (blue dotted line) with the parameters b_0=200, H_0=70, n=6, s = 5, γ = 0.003, ω_m = 0.0001, Ω_{m0} = 0.25

the Friedmann equation's left side, enabling it to clarify multiple scenarios of modified gravity. This model simply reduces to GR when $\beta_1 = 0$ and $\beta_2 = 1/2$. Taking the power-law form of a the energy density and pressure are obtained as

$$\rho_Q = 3s^2 b_0^{2/s}(z+1)^{2/s}\left(1 - \frac{(\sqrt{2}\beta_1)\left(\frac{\log\left(\frac{\sqrt{6}}{sb_0^{1/s}(z+1)^{1/s}}\right)+1}{b_0^{1/s}(z+1)^{1/s}} - \log\left(\sqrt{6}sb_0^{1/s}(z+1)^{1/s}\right)\right)}{\sqrt{3}s}\right) \quad (33)$$

$$p_Q = 3s^2 b_0^{2/s}(z+1)^{2/s}\Big[\sqrt{6}\beta_1 b_0^{-1/s}(z+1)^{-1/s}\left(1 - 2\log\left(sb_0^{1/s}(z+1)^{1/s}\right)\right)$$

$$-\frac{\sqrt{2}\beta_1\left[2\log\left(\frac{1}{sb_0^{1/s}(z+1)^{1/s}}\right)+1\right]}{\sqrt{3}sb_0^{1/s}(z+1)^{1/s}} - 2\beta_2(6s+1) - 3s + 2\Big] \quad (34)$$

Using these in eqns.(23) and (24) and also using eqn.(19) we get the expressions for $X(z)$ and $Y(z)$ for this model as,

$$X(z) = \frac{s^2 b_0^{2/s}(z+1)^{2/s}}{H_0^2}\left(1 - \frac{\sqrt{2}\beta_1}{\sqrt{3}s}\left(\frac{\log\left(\frac{\sqrt{6}}{sb_0^{1/s}(z+1)^{1/s}}\right)+1}{b_0^{1/s}(z+1)^{1/s}} - \log\left(\sqrt{6}sb_0^{1/s}(z+1)^{1/s}\right)\right)\right) + \Omega\mathrm{m0}(z+1)^{3(\omega\mathrm{m}+1)} \quad (35)$$

$$Y(z) = -\frac{1}{6H_0^2}\left(sb_0^{2/s}(z+1)^{2/s}\left(-2\sqrt{6}\beta_1 \log\left(sb_0^{1/s}(z+1)^{1/s}\right) - \sqrt{6}\beta_1 \log(6) + 6s(6\beta_2 + 9(4\beta_2+1)s - 7)\right) + \sqrt{6}\beta_1 sb_0^{1/s}(z+1)^{1/s}\right.$$

$$\left.\left(36s\log\left(sb_0^{1/s}(z+1)^{1/s}\right) + 14\log\left(\frac{b_0^{-1/s}(z+1)^{-1/s}}{s}\right) - 18s + 8 + \log(6)\right) - 6H_0^2(3\omega_m+1)\Omega_{m0}(z+1)^{3\omega_m+3}\right) \quad (36)$$

Using these two expressions in equation (25) we get a differential equation of η(z) for model-II. The trajectory for η(z) is given in Fig.(2). In the figure, the evolution of gravitational waves for this DGP-like f(Q) model is obtained. The plot is generated for two different values of ξ. The figure shows that we get gravitational waves of lower amplitude for a higher value of ξ. Moreover, with the evolution of time (as z becomes smaller) the waves proceed with decreasing amplitude.

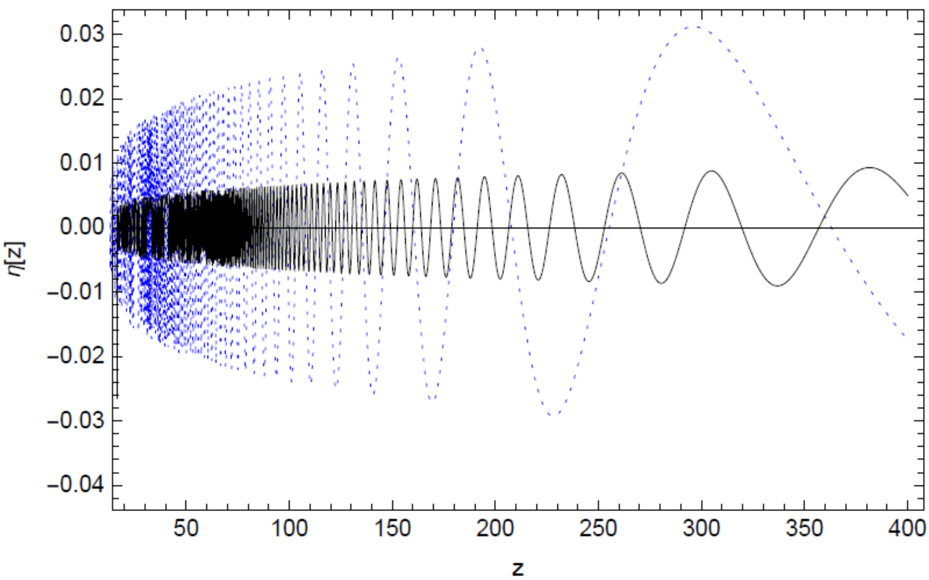

FIG. 2: The figure shows the variation of $\eta(z)$ against redshift z, for model II. Here we have ξ = 5×1011 (black line) and ξ = 2×109 (blue dotted line) with the parameters b_0=200, H_0=70, n=6; s = 5, ω_m = 0.0001, Ω_{m0} = 0.25,$\beta 1$ = 20,$\beta 2$ = 25

IV CONCLUSION

In this paper, we investigated the evolution of gravitational waves in the realm of *f(Q)* gravity. Two different models of *f(Q)* gravity are considered and the evolution of gravity waves is investigated. The basic idea is to examine the consequences of the non-metricity in the progression of gravity waves. To start in this direction, we looked at the flat Friedmann-Robertson-Walker (FRW) spacetime in *f(Q)* gravity. We examined energy densities for both dark energy and matter by examining distinct conservation equations for each component. We developed the perturbation equations regulating gravitational wave evolution with respect to redshift z in the FRW Universe backdrop using the field equations. The properties of gravitational waves for the *f(Q)* gravity model were then discussed. Because the gravitational wave differential equations in this model are complicated, we used graphical analysis to derive wave curves at different redshift ranges (Figs. 1 and 2). The amplitudes diminish with time as $z \to 0$, as shown in both the figures.

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